

Robust Control of Surge Instability in a Centrifugal Compressor

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Abstract- This paper proposes a robust design scheme for the control of surge phenomenon appearing in a centrifugal compressor. It is known the surge instability in compressor might lead to reduce the operating efficiency and even damage the machinery while both of unstable compressor characteristics and compressor load torque are hard to exactly measure in the practical application. Based on the assumption of local sector nonlinearity for system dynamics, a robust control law is proposed to cover the system nonlinearity and uncertainties while guaranteeing the stability of system equilibrium. This is achieved by using the actuation of the driving torque and closed-couple valve. Numerical simulations are given to demonstrate the success of the proposed robust controller design.

Keywords- Robust control; Centrifugal compressor; Surge phenomenon; Compressor characteristic; Local sector nonlinearity

I. INTRODUCTION

Over the past several years, the compression systems have been widely applied to civil and industrial applications such as air conditioner, automotive engines, aerospace and thermal plasma torch (e.g., [1], [2]). Among those applications, the compressors might be operated in the high pressure rise regarding the performance and efficiency requirements. Surge and rotation stall are two basic types of instability, which are known to appear in axial compression systems and might reduce the operating efficiency and even damage the machinery (e.g., [3]-[7]). In order to study those instabilities, Greitzer derived a nondimensional axial flow compressor dynamical model from which surge and rotation stall can be predicted [3]. Moore and Greitzer employed the Galerkin method to construct a third-order model for the study of rotating stall [4]. Among the existing studies, several control strategies have been proposed for the control of unstable instabilities in an axial compressor by adopting the Moore-Greitzer model (e.g., [5]-[7]). The axial flow compression system described by [4] is known to be uncontrollable (e.g., [5], [7]). Liaw and Abed proposed a local bifurcation analysis and bifurcation control to stabilize the instabilities of axial flow compressor [5]. In the study of [5], the existence conditions of surge and rotation stall were found by applying local bifurcation analysis. To further study the prevention of the unstable dynamics, Liaw and Abed employed the active nonlinear control to transform the subcritical bifurcation into a supercritical one. In addition, Chaturvedi and Bhat presented the output feedback backstepping controller that combined a high gain observer to estimate the mass flow [6]. Based on the dynamical characteristics of practical system, the unstable branch of compression characteristic is difficultly measured so that there exists an uncertainty term in an axial flow compressor. Liaw and Huang proposed a robust control

strategy to stabilize instability via sliding mode design with respect to the uncertainty appearing in the axisymmetric characteristics maps [7].

The study of a centrifugal compression system has attracted considerable attention in the recent years (e.g., [8]-[13]). Leonessa et al proposed a lumped-parameter surge model for a centrifugal compressor that involves the state of spool speed, pressure, and mass flow rate [8]. In order to prevent the surge dynamics, a switching type of globally stabilizing controller was also applied to control the unstable dynamics in a centrifugal compressor. In the study of [9], Gravdahl and Egeland developed an approximate compressor characteristics which considers the losses of incidence, friction, back flow, clearance etc. Besides, a surge control law was designed for variable speed operation of centrifugal compressor via PI speed control [9]. Bohagen and Gravdahl presented the surge control scheme with respect to driving torque as a control input via backstepping and passivity approaches [10]. In addition, the centrifugal compressor might exist the uncertainty term in the characteristic maps, which are similar to that in axial flow compression system.

For the study of centrifugal compressor with system uncertainty, the robust control of surge behaviour has been proposed in [11]-[13]. Darogheh et al employed a high gain robust adaptive controller to suppress the surge dynamics of centrifugal compression system with variable speed [11]. In that investigation [11], non-dimensional time varying disturbance rejections are considered in compressor surge control. The control schemes in [12] were based on the backstepping method to achieve global stability. In order to compensate the system uncertainty, robust global stability via the Lyapunov redesign technique was employed in such a circumstance [12]. In addition, Shehata et al provided an active control in a constant speed centrifugal compression system which is simplified to the second order model [13]. That study employed a different type of controller based on the specific requirements of performance. Among those studies, robust control schemes for centrifugal compressor have been proposed to deal with uncertainty or disturbance. However, the robust design proposed in those studies requires the bounds of system uncertainty which might not be easily obtained in practical applications.

In this paper, we consider different approach by using sector nonlinearity approach. The modelling error and/or measurement error of compressor's driving torque and characteristic maps will be treated as system uncertainties. According to the study of [7], Liaw and Huang utilized an

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“additive-type” control to the mass flow of axial flow compressor such that the domain of attraction for stable equilibrium was enlarged via sliding mode control. Motivated by [7], one of the main goals of this paper is to enlarge the domain of attraction for providing the non-local stability of system equilibrium in centrifugal compressor dynamics. By such a design, the system uncertainty will then be compensated for guaranteeing system stability. In this paper, we try to use the local sector boundary to cover the nonlinear term in the compressor system. The local sector nonlinearity guarantees that nonlinear terms of the system can be bounded by a linear function (e.g., [14]-[15]).

This paper is organized as follows. In Section II, we recall the third-order nonlinear lumped-parameter model for centrifugal compression system of which the instabilities are described. The robust control schemes are derived in Section III. Section IV reveals the numerical simulations which demonstrate the feasibility of the proposed control laws. Finally, the conclusions are summarized in Section V.

II. DYNAMICAL BEHAVIOUR OF A CENTRIFUGAL COMPRESSOR

A centrifugal compressor is known to be consisted of inlet duct, outlet duct, plenum, exit duct, valve, and compressor motor as depicted in Fig. 1. Detailed modelling process through instrumentation and experimental results of centrifugal compressor can be referred to (e.g., [8], [9], [16]). A nonlinear lumped-parameter third-order equation by using notation of [12] is recalled as follows:

$$J \frac{d\omega}{dt} = \tau_t - \tau_c \quad (1)$$

$$\frac{L}{A} \frac{d\dot{m}_c}{dt} = P_c - P_p \quad (2)$$

$$\frac{dP_p}{dt} = \frac{a^2}{V_p} (\dot{m}_c - \dot{m}_t), \quad (3)$$

where ω denotes spool angular speed while τ_t and τ_c are the drive torque and compressor load torque, respectively. J denotes the spool moment of inertia. P_c is the pressure downstream of the compressor. P_p is the pressure within the plenum. L is the length between the compressor and duct while A is a reference area. $\dot{m}_t$ and $\dot{m}_c$ denote the mass flow rate through the throttle and mass flow rate at the plenum entrance, respectively. In addition, a denotes the inlet stagnation sonic velocity and V_p is the plenum volume. Note that, for the case of the spool speed of centrifugal compressor being set to a constant value, the system (1)-(3) is found to be equivalent to the model derived by Greitzer (1976) [3].

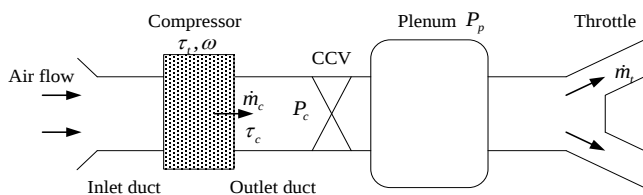


Fig. 1 A Schematic Diagram of Centrifugal Compression System

It is known that (e.g., [8], [9], [16]) that the compressor load torque is a function of the spool speed and the mass flow rate. The system (1)-(3) can be rewritten in non-dimensional forms as given below

$$\dot{\tilde{\omega}} = \bar{a}[\tau_t - \Gamma(\tilde{\omega}, \phi)] \quad (4)$$

$$\dot{\phi} = \bar{b}[\psi_c(\tilde{\omega}, \phi) - \psi] \quad (5)$$

$$\dot{\psi} = \bar{c}[\phi - F(\gamma, \psi)], \quad (6)$$

where $\Gamma(\phi, \tilde{\omega})$ and $\psi_c(\tilde{\omega}, \phi)$ are defined as the compressor load torque function and the compressor characteristic map, respectively. It is observed from equations (4)-(6) that the nonlinear terms mainly originate from compressor load torque $\Gamma(\tilde{\omega}, \phi)$, characteristic maps $\psi_c(\tilde{\omega}, \phi)$ and throttle opening characteristic function $F(\gamma, \psi)$. The dynamics of load torque and characteristic map in compressor are known to depend on the effects of mass flow and spool speed. In addition, $F(\gamma, \psi)$ is generally a strictly increasing function with respect to both of the pressure rise and the throttle opening γ . Moreover, in practical application, we have $\bar{a} > 0$, $\bar{b} > 0$, and $\bar{c} > 0$, which are the non-dimensional parameters defined in (e.g., [8], [9], [16]). Denote $x_e = (\tilde{\omega}_e, \phi_e, \psi_e)^T$ an equilibrium point of systems (4)-(6). It is known from [12] that the equilibrium point x_e of systems (4)-(6) will be asymptotically stable if $\frac{\partial \psi_c(\tilde{\omega}, \phi)}{\partial \phi}|_{x_e} < 0$ but being unstable if $\frac{\partial \psi_c(\tilde{\omega}, \phi)}{\partial \phi}|_{x_e} > 0$. Normally, when a compressor operates near the surge line, a small variation of system parameter might cause the compressor to exhibit an oscillative behaviour which is found to be related to compressor characteristic maps [12]. Unfortunately, the unstable compressor characteristics are hard to measure in practical applications. The curve fitting scheme is usually used to approximate the unstable characteristic maps. Such an approach has been used in [8] for obtaining the characteristic maps of centrifugal compressor with respect to different setting value of spool speed. One example is shown in Fig. 2.

The bifurcation diagrams for system (4)-(6) with respect to the variation of compressor torque τ_t adopted from ([8], [12]) are shown in Fig. 3, where the notations HB and PD denote the Andronov-Hopf bifurcation point and period-doubling bifurcation point, respectively. In addition, the solid line (resp. open circle) denotes the stable equilibrium branch (resp. stable limit cycle) while dot line (resp. solid circle) denotes unstable equilibrium branch (resp. unstable limit cycle). It is observed from Fig. 3 that the system states exhibit a stable limit cycle when the compressor operates at the equilibrium point near the bifurcation point HB. The timing responses from numerical simulations are shown in Fig. 4 to demonstrate such a behaviour. Here, we choose $\tau_t = 0.1$ and $\gamma = 0.315$ (i.e., $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.557403, 0.199432, 0.4008)$) with initial value $(\tilde{\omega}_0, \phi_0, \psi_0) = (0.5, 0.345, 0.22)$. The oscillative behaviour depicted in Fig. 4 is known to be the so-called “surge” instability.

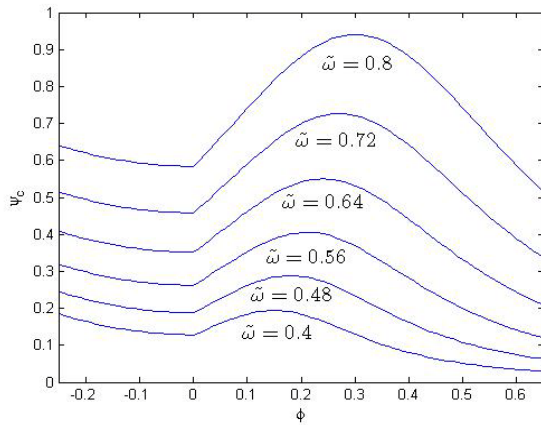


Fig. 2 Characteristic Maps of a Centrifugal Compression System

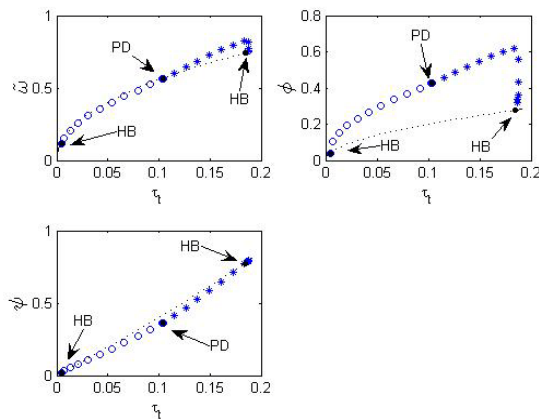


Fig. 3 Bifurcation Diagrams with Respect to the Variation of Driving Torque

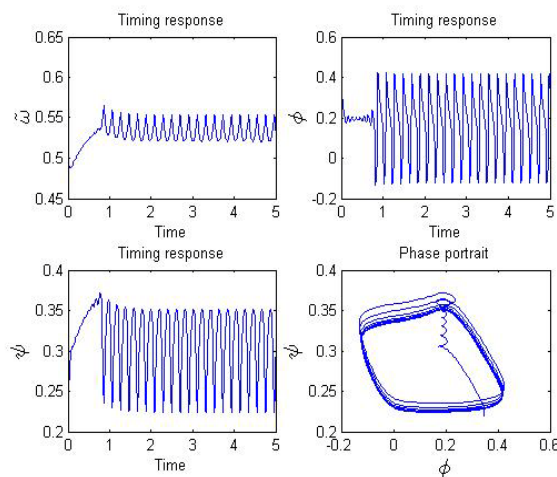


Fig. 4 Timing Responses and Phase Portraits

It is shown in [17] that the variation of the driving torque τ_t and/or the throttle opening γ might cause a centrifugal compressor to exhibit surge dynamics. In addition, the domain of attraction for the system equilibrium points is found in [17] to become smaller as the compressor drive torque τ_t decreases. Due to the effects of uncertainty in $\Gamma(\tilde{\omega}, \phi)$ and $\psi_c(\tilde{\omega}, \phi)$, those will result in the system running into the regime of HB.

Thus, the surge instability might be avoided by making the domain of attraction large enough. In this paper, a surge recovery control schemes is employed to drive system dynamics toward a stable equilibrium by using compressor drive torque and the actuation of closed-couple valve. Besides, the proposed control scheme will also enlarge the domain of attraction for the stable equilibrium point, which will then keep the system behaviour in the stable manifold disregard the appearance of disturbance and/or system uncertainty.

III. ROBUST CONTROL SCHEME

In this section, we will propose robust control laws to stabilize the compressor dynamics with system uncertainty by using the actuation of compressor driving torque and close-coupled valve. Let $x_e = (\tilde{\omega}_e, \phi_e, \psi_e)^T$ be the equilibrium point of system (4)-(6) for the given driving torque $\tau_t = \tau_{te}$ and throttle opening $\gamma = \gamma_e$. Here, we consider the control inputs of the compressor will be the compressor driving torque and the close-coupled valve.

Let $x = (x_1, x_2, x_3)^T$ with $x_1 = \tilde{\omega} - \tilde{\omega}_e$, $x_2 = \phi - \phi_e$, $x_3 = \psi - \psi_e$, $u_1 = \tau_t - \tau_{te}$ and u_2 denotes the actuation of the close-coupled valve. The system (4)-(6) can then be rewritten as follows:

$$\dot{x}_1 = \bar{a}[f(\tau_{te}, x_1, x_2) + u_1] \quad (7)$$

$$\dot{x}_2 = \bar{b}[C_{ss}(x_1, x_2) - x_3 + u_2] \quad (8)$$

$$\dot{x}_3 = \bar{c}[x_2 + g(\gamma_e, x_3)], \quad (9)$$

where

$$f(\tau_{te}, x_1, x_2) = \tau_{te} - \Gamma(x_1 + \tilde{\omega}_e, x_2 + \phi_e) \quad (10)$$

$$C_{ss}(x_1, x_2) = \psi_c(x_1 + \tilde{\omega}_e, x_2 + \phi_e) - \psi_c(\tilde{\omega}_e, \phi_e) \quad (11)$$

$$g(\gamma_e, x_3) = F(\gamma_e, \psi_e) - F(\gamma_e, x_3 + \psi_e) \quad (12)$$

with the following equilibrium conditions for system (4)-(6):

$$\tau_{te} = \Gamma(\tilde{\omega}_e, \phi_e) \quad (13)$$

$$\phi_e = F(\gamma_e, \psi_e) \quad (14)$$

$$\psi_e = \psi_c(\tilde{\omega}_e, \phi_e) \quad (15)$$

The main goal of this paper is to design control laws by using control inputs u_1 and u_2 to compensate the surge instability and enlarge the domain of attraction for the stabilized system equilibrium. Detailed are given as follows.

A. System Uncertainty for a Centrifugal Compressor

In the practical application, the stable portion of the dynamical characteristics for a centrifugal compressor can easily be measured from the experimentation. On the contrary, the unstable behaviour might be difficult to obtain from the practical compression system. Due to the fact of the inexact measurement, it is unavoidable to have the appearance of the uncertain terms in the compressor dynamics for practical usage. In addition to compressor characteristic maps, the uncertainty in compressor load torque might also emerge from the fluctuations of the unstable spool angular speed and mass flow rate. Thus, both compressor load torque and characteristic

maps are required to consider the existence of uncertain terms in the robust controller design. Let the compressor torque and characteristic map are given as (e.g., [12])

$$f(\tau_t, x_1, x_2) = \bar{f}(\tau_t, x_1, x_2) + \Delta f(x_1, x_2) \quad (16)$$

$$C_{ss}(x_1, x_2) = \bar{C}_{ss}(x_1, x_2) + \Delta C_{ss}(x_1, x_2), \quad (17)$$

where $\bar{f}(\tau_t, x_1, x_2)$ and $\bar{C}_{ss}(x_1, x_2)$ denote nominal models while $\Delta f(x_1, x_2)$ and $\Delta C_{ss}(x_1, x_2)$ are the uncertainties, respectively.

In the following, we will discuss those two uncertainties. Details are given below.

1) Uncertainty in Compressor Torque:

Based on the conservation of angular momentum in spool dynamics, the compressor load torque is required to overcome the duct wall friction [16]. For a compression system, Euler's turbine equation is used to describe the compressor torque which depends on the angular momentum of the fluid [18]. The schematic compressor load torque is adopted from [18] as shown in Fig. 5. As a result of turbo-machine driving at an angular speed, the compressor torque can be obtained by Euler's turbine equation as given by (e.g., [9], [18]):

$$\tau_c = \phi(r_2 v_{\theta 2} - r_1 v_{\theta 1}), \quad (18)$$

where r_1 and r_2 are radii at the inducer tip and the inducer hub casing, respectively. In addition, the notation $v_{\theta 1}$ denotes instantaneous tangential velocity of which the swirling fluid passes through the inducer tip at radius r_1 while $v_{\theta 2}$ is the tangential velocity at inducer hub casing with radius r_2 . However, the various power losses might exist in the practical system such as incidence losses, fluid friction losses and inlet casing losses [9]. Those energy losses might procure the fluctuation behaviour into the compressor torque. Hence, such kinds of uncertainty and disturbance need to be considered in the dynamical model of a centrifugal compressor.

In order to compensate the uncertainty in compressor torque, we introduce the sector nonlinearity method to guard the operating boundary of compressor torque. It is observed from Eq. (18) and Fig. 5 that the compressor's driving torque is associated with mass flow rate and spool angular speed. Based on the above discussion, we make the following assumption.

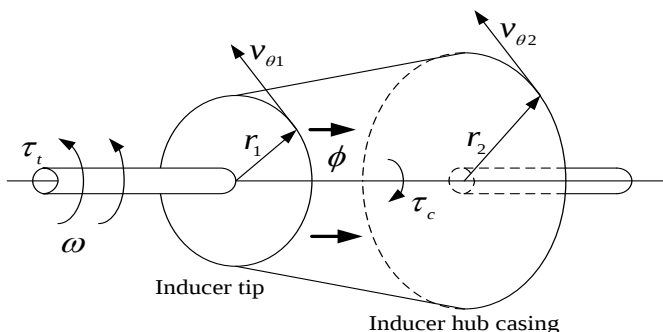


Fig. 5 Compressor Torque in Turbo-Machine

Assumption 1: Suppose that the compressor's driving torque with uncertainty satisfies the following condition:

$$|\bar{f}(\tau_t, x_1, x_2) + \Delta f(x_1, x_2)| \leq M_1 |x_1| + M_2 |x_2| \quad (19)$$

for all x_1 and x_2 in $D \in \mathbb{R}^2$, where D denotes an open set containing the origin.

Note that, $M_1 |x_1|$ and $M_2 |x_2|$ are apparently the allowed local sector bounds for the compressor's driving torque function.

2) Uncertainty in Compressor Characteristic Maps:

It is known that the stable operation region of the compressor characteristic can be collected from practical experiment. However, the unstable behaviour of compressor dynamics is hardly measured. Based on the structure of the centrifugal compressor, inexact unstable branch might cause the existence of uncertainty in the compressor characteristic maps. Thus, it is inevitable that such an uncertainty requires to be considered in the design of robust control scheme. For the centrifugal compression system, the characteristic map depends on the mass flow rate and spool angular speed. We have the next assumption for characteristic map.

Assumptions 2: Suppose that the compressor characteristic map with uncertainty satisfies the following condition:

$$|\bar{C}_{ss}(x_1, x_2) + \Delta C_{ss}(x_1, x_2)| \leq N_1 |x_1| + N_2 |x_2| \quad (20)$$

for all x_1 and x_2 in $D \in \mathbb{R}^2$, where D denotes an open set containing the origin.

Note that, $N_1 |x_1|$ and $N_2 |x_2|$ in Assumption 1 above denote the local sector bounds for compressor characteristic map.

B. Robust Control Design

To fulfill the stabilization design for the centrifugal compressor dynamics, we employ the robust control scheme to carry out the suppression of surge phenomenon. According to Assumptions 1 and 2, the nonlinear terms of the system with the appearance of uncertainties in both of compressor's driving torque and characteristic map are assumed to be limited in local sector bounds. Furthermore, it is clear that the system (7)-(9) is controllable. In this paper, we employ the local sector approach to design the robust controller for the system (7)-(9) with both driving torque u_1 and close-couple valve u_2 .

Let

$$V(x) = \frac{1}{2} \left(\frac{1}{a} x_1^2 + \frac{1}{b} x_2^2 + \frac{1}{c} x_3^2 \right) \quad (21)$$

be a Lyapunov function candidate for system (7)-(9). The time derivative of the Lyapunov function candidate is given by

$$\begin{aligned} \dot{V}(x) &= \frac{1}{a} x_1 \dot{x}_1 + \frac{1}{b} x_2 \dot{x}_2 + \frac{1}{c} x_3 \dot{x}_3 \\ &= x_1 [\bar{f}(\tau_{te}, x_1, x_2) + \Delta f(x_1, x_2) + u_1] \\ &\quad + x_2 [\bar{C}_{ss}(x_1, x_2) + \Delta C_{ss}(x_1, x_2) + u_2] \\ &\quad + x_3 [F(\gamma_e, \psi_e) - F(\gamma_e, x_3 + \psi_e)] \\ &\leq M_1 x_1^2 + x_1 u_1 + (M_2 + N_1) |x_1| |x_2| + N_2 x_2^2 \\ &\quad + x_2 u_2 + x_3 [F(\gamma_e, \psi_e) - F(\gamma_e, x_3 + \psi_e)] \end{aligned} \quad (22)$$

Assume F is a strictly increasing function with respect to x_3 . It is obvious to have that $x_3[F(\gamma_e, \psi_e) - F(\gamma_e, x_3 + \psi_e)] \leq 0$ for all x_3 while $x_3[F(\gamma_e, \psi_e) - F(\gamma_e, x_3 + \psi_e)] = 0$ hold at $x_3 = 0$ [12]. Let the control inputs u_1 and u_2 be chosen as

$$u_1 = -k_1 x_1 \quad (23)$$

and

$$u_2 = -k_2 x_2 \quad (24)$$

For $k_1 > 0$ and $k_2 > 0$. We then have

$$\begin{aligned} \dot{V}(x) &\leq (M_1 - k_1)x_1^2 + (M_2 + N_1)|x_1| \cdot |x_2| + (N_2 - k_2)x_2^2 \\ &= (M_1 - k_1)[|x_1| + \frac{(M_2 + N_1)|x_2|}{2(M_1 - k_1)}]^2 \\ &\quad + [N_2 - k_2 - \frac{(M_2 + N_1)^2}{4(M_1 - k_1)}]x_2^2 \end{aligned} \quad (25)$$

It is clear from the above discussions that $\dot{V}(x)$ in (25) will be a negative definite function, while both k_1 and k_2 are large enough. We then have the next results.

Theorem 1: Suppose that Assumptions 1 and 2 hold. The equilibrium points x_e of system (7)-(9) will be locally asymptotically stabilized by a linear control law given in (23) and (24) with k_1 and k_2 being large enough.

Note that, the proposed control scheme given in Theorem 1 can be achieved without having exact knowledge of the compressor characteristic map $C_{ss}(x_1, x_2)$ and compressor torque function $f(\tau_t, x_1, x_2)$. However, it does require to have the sector bound to cover the local behaviour of nonlinear terms with the uncertainties.

IV. NUMERICAL SIMULATIONS

In order to check the effectiveness of the robust controller design on system performance, we adopt the centrifugal compressor model with same parameter values from [7]-[8] for numerical study. Here, the compressor torque is given by

$$\Gamma(\tilde{\omega}, \phi) = \sigma |\phi| \tilde{\omega} \quad (26)$$

and throttle function

$$F(\gamma, \psi) = \gamma \sqrt{\psi}. \quad (27)$$

In addition, the compressor characteristic maps $\psi_c(\tilde{\omega}, \phi)$ is given by

$$\psi_c(\tilde{\omega}, \phi) = [1 + \sigma \eta_c(\tilde{\omega}, \phi) \bar{d} \tilde{\omega}^2]^{\frac{\gamma_c}{\gamma_c - 1}} - 1, \quad (28)$$

where

$$\eta_c(\tilde{\omega}, \phi) = \frac{\sigma \tilde{\omega}^2}{\sigma \tilde{\omega}^2 + \frac{1}{2}(f_1 \tilde{\omega} - f_2 \phi)^2 + \frac{1}{2}(\sigma \tilde{\omega} - f_3 \phi)^2 + \phi^2(f_4 + f_5)}. \quad (29)$$

The notations $\bar{d}$ and f_i , $i=1, \dots, 5$ are non-dimensional parameters, and σ denotes the slip factor. In the modeling of the centrifugal compressor, the characteristic map given in Eq. (29) holds at $\phi > 0$. For the case of deep surge involving negative mass flow, the compressor characteristic of deep surge for $\phi \leq 0$ is shown as ([7]-[8])

$$\psi_c(\tilde{\omega}, \phi) = \rho \phi^2 + \psi_{c0}(\tilde{\omega}, \phi), \quad (30)$$

where

$$\psi_{c0}(\tilde{\omega}, \phi) = (1 + \sigma \eta_{c0} \bar{d} \tilde{\omega}^2)^{\frac{\gamma_c}{\gamma_c - 1}} - 1 \quad (31)$$

and

$$\eta_{c0} = \frac{2\sigma}{\sigma^2 + 2\sigma + f_1^2}$$

The value of parameters are given as:

$$(\bar{a}, \bar{b}, \bar{c}, \bar{d}) = (6.97, 310.81, 9.37, 0.38)$$

$$(f_1, f_2, f_3, f_4, f_5) = (0.44, 1.07, 2.18, 0.17, 0.12)$$

$$\gamma_c = 1.4, \quad \rho = 3 \text{ and } \sigma = 0.9$$

To compare with the results shown in Fig. 3, for the following numerical simulation we choose $\gamma = 0.315$ with initial condition $(\tilde{\omega}_0, \phi_0, \psi_0) = (0.5, 0.345, 0.22)$.

A. Open-Loop Dynamics

Based on the Fig. 3, we choose a different equilibrium point to simulate the dynamical behaviour of the centrifugal compressor. The numerical simulations of timing responses and phase plane for the open-loop system are obtained by using code Matlab and are shown in Figs. 6-8. As depicted in Fig. 6, the system equilibrium $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.766608, 0.289736, 0.846)$ for $\tau_t = 0.1999$ is found to be asymptotically stable. The surge instabilities are observed in Figs. 7 and 8. In Fig. 7, we do observe that system behaviour reveals period-two oscillation for the system equilibrium $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.606924, 0.21964, 0.4864)$ when $\tau_t = 0.12$. In addition, the multi-period oscillation of centrifugal compressor dynamics is obtained for the equilibrium point $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.665538, 0.244524, 0.6026)$ when $\tau_t = 0.1465$. The corresponding timing responses and phase portrait are shown in Fig. 8.

Under above analysis of the uncontrolled centrifugal compressor model, the system might fall into a violent surge phenomenon with respect to the variation of driving torque and/or throttle opening. It can be observed from Figs. 6-8 that slight variation of the driving torque and/or the throttle opening might cause the surge dynamics to occur in the centrifugal compressor system.

B. Closed-Loop Dynamics

In the following, the robust control schemes developed in Section III are employed to stabilize the system behaviour which considers the system uncertainty in the centrifugal

compressor dynamics. The effectiveness of the proposed robust control law will also be demonstrated.

Consider the system (4)-(6) with control inputs u_1 and u_2 as given by

$$\dot{\tilde{\omega}} = \bar{a}[\tau_i - \Gamma(\tilde{\omega}, \phi) + u_1] \quad (32)$$

$$\dot{\phi} = \bar{b}[\psi_c(\tilde{\omega}, \phi) - \psi + u_2] \quad (33)$$

$$\dot{\psi} = \bar{c}[\phi - F(\gamma, \psi)]. \quad (34)$$

Here, the control inputs u_1 and u_2 denote driving torque and close-couple valve, respectively. According to Theorem 1, we have the control law as given by

$$u_1 = -k_1(\tilde{\omega} - \tilde{\omega}_e) \quad (35)$$

and

$$u_2 = -k_2(\phi - \phi_e), \quad (36)$$

where $\tilde{\omega}_e$ and ϕ_e denote the value of the corresponding unstable equilibrium point at which system might exhibit oscillative behaviour.

1) Controller Design without System Uncertainty:

First, we adopt the centrifugal compressor model without considering system uncertainty. The control inputs u_1 and u_2 given in (35) and (36) are used for system (32)-(34) without considering system uncertainty. It is clear to have the bounds for the functions $f(\tau_i, x_1, x_2)$ and $C_{ss}(x_1, x_2)$ to meet Assumptions 1 and 2. For the control design, we choose $k_1 = 0.44$ and $k_2 = 0.59$. The parameters for the numerical study are obtained for three different equilibrium points $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.557403, 0.199432, 0.4008)$, $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.606924, 0.219694, 0.4864)$ and $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.665538, 0.244524, 0.6026)$. As depicted in Figs. 9-11, the timing trajectories of system states are pushed toward the system equilibrium by the control inputs u_1 and u_2 . As depicted in Figs. 4 and 9, the system state is clearly pushed from period-one oscillation to a stable operating point by control inputs u_1 and u_2 . Compared with the surge dynamics depicted in Fig. 7, it can be observed that the period-two behaviour is stabilized to the equilibrium point as shown in Fig. 10. Similarly, the complicated dynamics depicted in Fig. 8 is also stabilized by the combination of the driving torque and close-coupled valve control as shown in Fig. 11. In addition, as depicted in Fig. 12, we consider that the controller is switched at time $t = 5$. It is observed from Fig. 12 that the system instabilities prior to the control action are diminished after adding control effort.

2) Controller Design with System Uncertainty:

Next, we present the numerical results of the system (7)-(9) with the robust control law. Following the control design procedure proposed in Section III, the control inputs u_1 and u_2 are used to compensate the system uncertainties embedded in

compressor's driving torque and characteristic map. The system (7)-(9) can then be rewritten as

$$\dot{x}_1 = \bar{a}[\bar{f}(\tau_i, x_1, x_2) + \Delta f(x_1, x_2) + u_1] \quad (37)$$

$$\dot{x}_2 = \bar{b}[\bar{C}_{ss}(x_1, x_2) + \Delta C_{ss}(x_1, x_2) - x_3 + u_2] \quad (38)$$

$$\dot{x}_3 = \bar{c}[x_2 + g(\gamma_e, x_3)] \quad (39)$$

Motivated by [12], the system uncertainties are chosen as

$$\Delta f(x_1, x_2) = 0.1 \sin(10x_2) \quad (40)$$

$$\Delta C_{ss}(x_1, x_2) = 0.1 \sin(10x_2). \quad (41)$$

The characteristic maps of centrifugal compressor with system uncertainty are shown in Fig. 13. In those figures, the dot line denotes the characteristic maps with uncertainty and the solid line is the nominal one. It is not difficult to find the bounds for the functions $f(\tau_i, x_1, x_2) + \Delta f(x_1, x_2)$ and $C_{ss}(x_1, x_2) + \Delta C_{ss}(x_1, x_2)$ to meet the Assumptions 1 and 2.

First, we use the control gain $k_1 = 0.44$ and $k_2 = 0.59$ to stabilize the spool dynamics of the system (37)-(39) with the addition of system uncertainties. Numerical simulations are shown in Fig. 14. It is observed from Fig. 14 that those control gains can not compensate the system uncertainties for the equilibrium point $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.665538, 0.244524, 0.6026)$. The system dynamics still exhibits oscillative behaviour. Based on the bounds for the uncertainties (40) and (41), we choose the control gain to be larger. That is, to let $k_1 = 0.5$ and $k_2 = 1.3$. The simulation results depicted in Fig. 15 show that the oscillative dynamics is stabilized by using the larger gains of control k_1 and k_2 . As shown in Fig. 16, a switching control scheme is added to demonstrate the effectiveness of the proposed robust control law. The control will be applied to system at time $t = 5$. It is observed that the system instabilities can be diminished by robust controller. The numerical simulations are found to agree with the analytical results of Theorem 1.

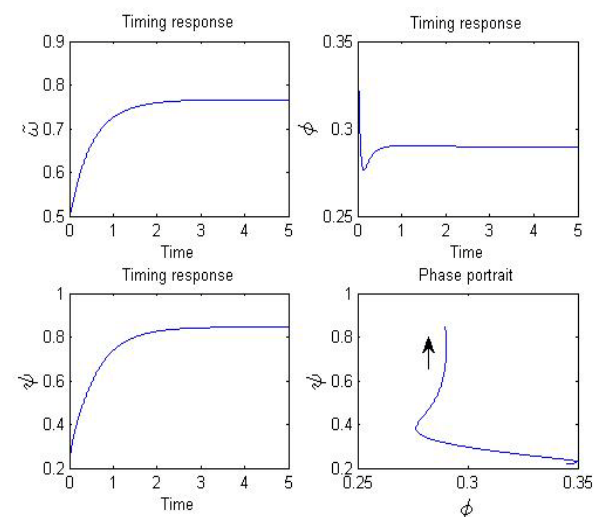


Fig. 6 Dynamical Behaviour in Open-Loop System for Equilibrium Point $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.766608, 0.289736, 0.846)$

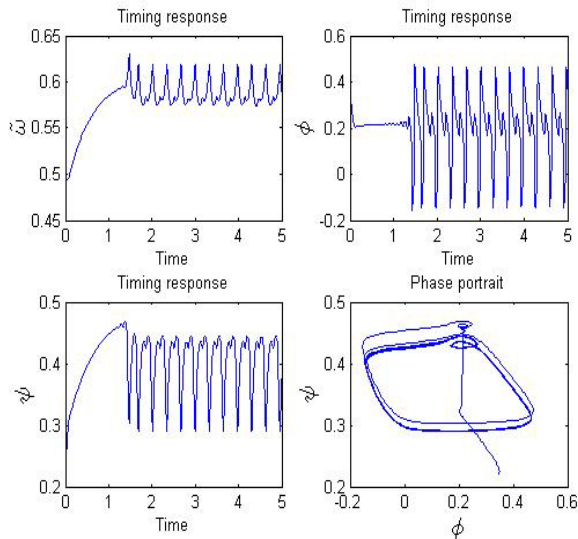


Fig. 7 Dynamical Behaviour in Open-Loop System for Equilibrium Point
 $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.606924, 0.2197, 0.4864)$

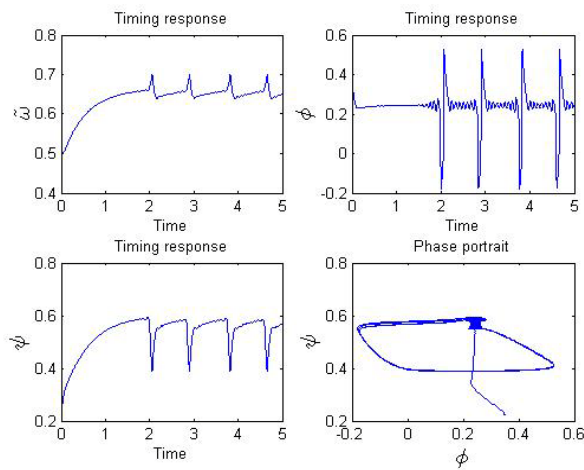


Fig. 8 Dynamical Behaviour in Open-Loop System for Equilibrium Point
 $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.665538, 0.244524, 0.6026)$

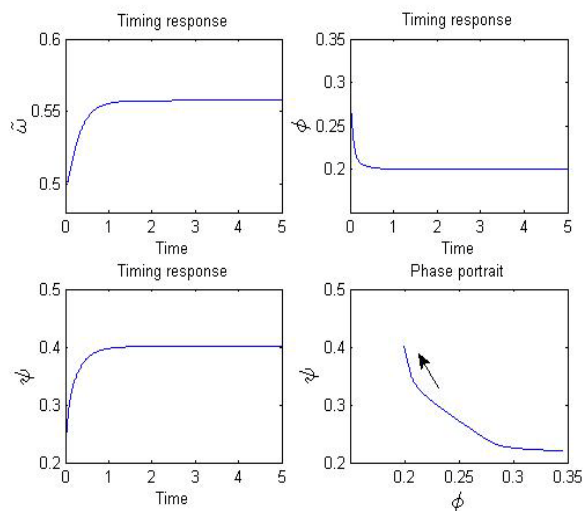


Fig. 9 Dynamical Behaviour in Closed-Loop System for Equilibrium Point
 $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.557403, 0.199432, 0.4008)$

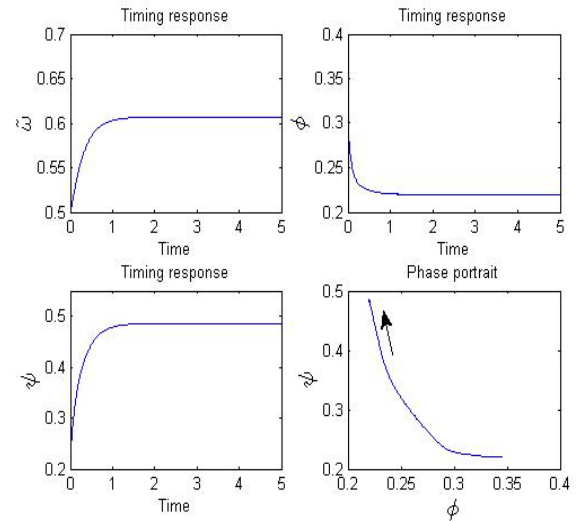


Fig. 10 Dynamical Behaviour in Closed-Loop System for Equilibrium Point
 $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.606924, 0.219694, 0.4864)$

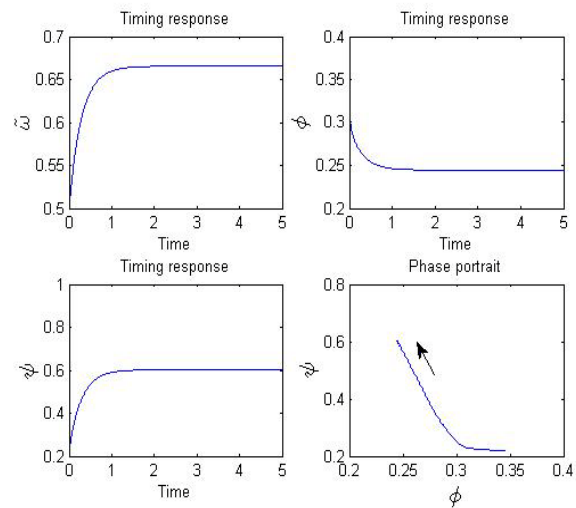


Fig. 11 Dynamical Behaviour in Closed-Loop System for Equilibrium Point
 $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.665538, 0.244524, 0.6026)$

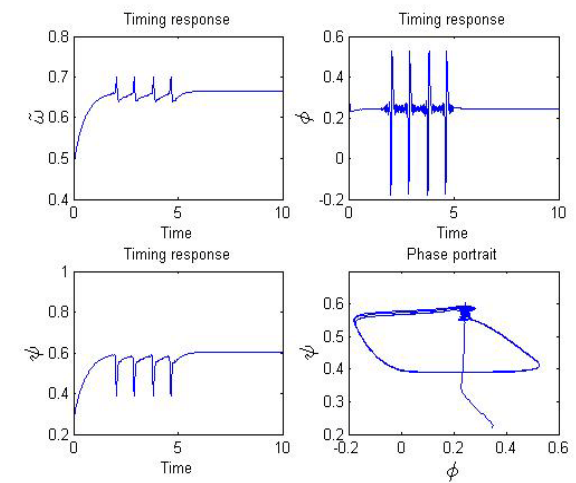


Fig. 12 Dynamical Behaviour with Control Switched at $t=5$ for Equilibrium Point
 $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.665538, 0.244524, 0.6026)$

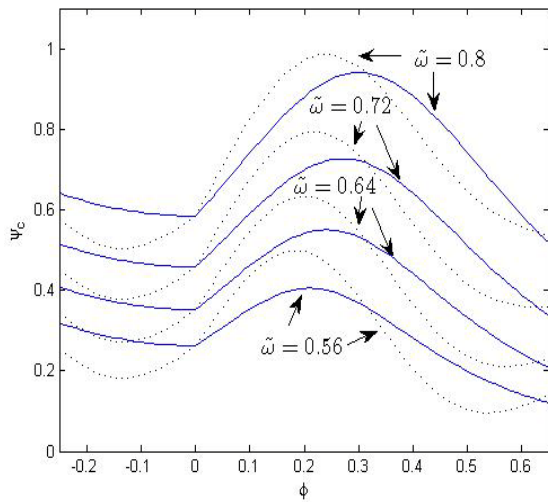


Fig. 13 Compare with the Uncertainty in the Characteristic Maps

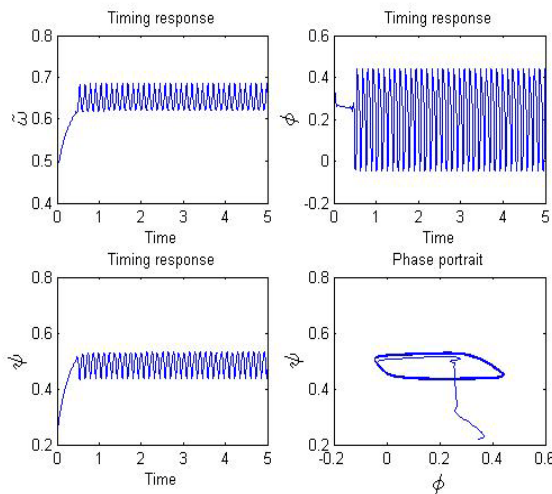


Fig. 14 Closed-Loop System with Uncertainties for Equilibrium Point $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.665538, 0.244524, 0.6026)$

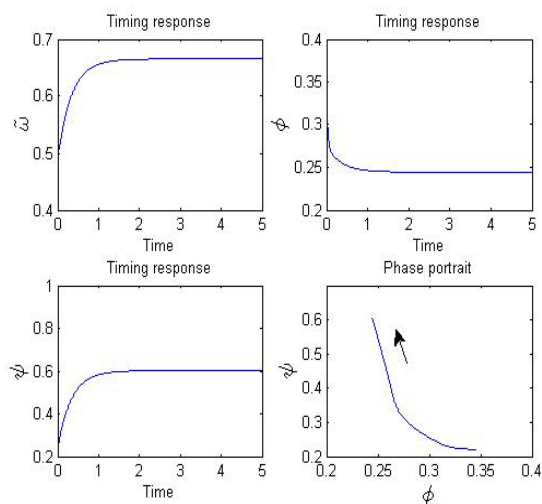


Fig. 15 Dynamical Behaviour with Robust Control Scheme for Equilibrium Point $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.665538, 0.244524, 0.6026)$

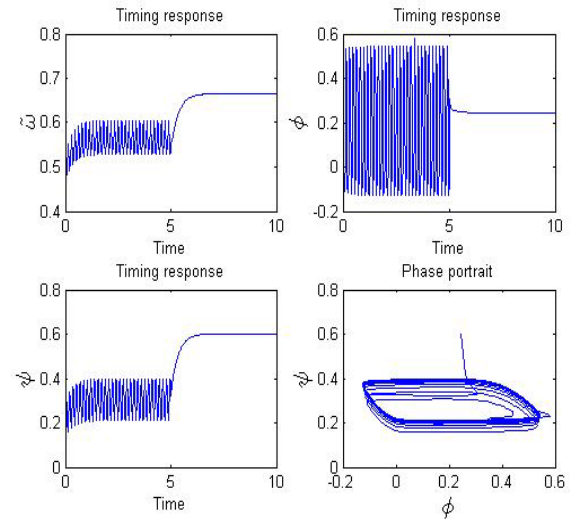


Fig. 16 Dynamical Behaviour with Robust Control Scheme for Equilibrium Point $(\tilde{\omega}_e, \phi_e, \psi_e) = (0.665538, 0.244524, 0.6026)$

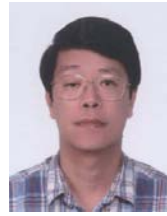
CONCLUSIONS

A robust control design is proposed in this paper for the control of surge phenomenon in centrifugal compressor dynamics via local sector nonlinearity approach. The proposed control scheme was shown to be robust with the appearance of system uncertainties embedded in the unstable operation region of centrifugal compressor. In addition, the proposed robust controller design can achieve the system stabilization without having exact knowledge of characteristic map function and compressor's driving torque function. Based on such an advantage, the proposed control design might provide an implementable strategy in the practical applications.

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